CONTENTS

PREFACE **v**

0 INTRODUCTION TO STOCHASTIC PROCESSES **1**

0.1 Stochastic processes and their characterization 4
0.2 A pivotal characteristic of some stochastic processes 13
0.3 A few examples of stochastic processes 21

1 POISSON PROCESSES **39**

1.1 Properties of the exponential distribution 42
1.2 Poisson process: three definitions 56
1.3 Event times in Poisson processes 68
1.4 Merging and splitting Poisson processes 81
1.5 Non-homogeneous Poisson process 95
1.6 Conditional Poisson process 112
1.7 Compound Poisson process 120

2 RENEWAL PROCESSES **129**

2.1 Introduction 132
2.2 Properties of the number of renewals 134
2.3 Renewal function 140
2.4 Renewal-type equations 150
2.5 Key renewal theorem and some other limit theorems 161
2.6 Recurrence times; the inspection paradox 182
2.7 Renewal reward processes 201
2.8 Alternating renewal processes 214
2.9 Delayed renewal processes 221
2.10 Regenerative processes 231

3 DISCRETE-TIME MARKOV CHAINS 239

3.1 Definitions and examples 242
3.2 Chapman-Kolmogorov equations; marginal and joint distributions 247
3.3 Classification of states; recurrent and transient states 258
3.4 Asymptotic behaviour of irreducible Markov chains 280
3.5 Asymptotic behaviour of reducible Markov chains 292
3.6 Markov chains with costs/rewards 295
3.7 Reversible Markov chains 304
3.8 Branching processes 314
3.9 First passage times; absorption probabilities 334

4 CONTINUOUS-TIME MARKOV CHAINS 347

4.1 Definitions and examples 351
4.2 Properties of the transition matrix; Chapman-Kolmogorov equations 354
4.3 Computing the transition matrix: finite state space 367
4.4 Birth and death processes 369
4.5 Classification of states 387
4.6 Asymptotic behaviour 393
4.7 Birth and death queueing systems in equilibrium 405
4.7.1 Description of a queueing system 405
4.7.2 Performance measures 409
4.7.3 M/M/1, the single server queueing system 416
4.7.4 M/M/∞, the queueing system with responsive servers 430
4.7.5 M/M/m, the m–server queueing system 439
4.7.6 M/M/m/m, the m–server loss queueing system 452

BIBLIOGRAPHY 463

HYPERLINKS 473

ACRONYMS 477

INDEX 479

FORMULAE 485